Testing Non-nested Systems of Factor Demand Equations

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Abstract: Empirical factor demand analysis typically involves making a choice from among several competing non-nested functional forms. Each of the commonly used factor demand systems, such as Translog, Generalized Leontief, Quadratic, and Generalized McFadden, can provide a valid and useful empirical description of the underlying production structure of the firm. As there is no theoretical guidance on selecting the model which is best able to capture the relevant features of the data, formal testing procedures can provide additional information. Paired and joint univariate non-nested tests of a null model against both single and multiple alternatives have been discussed at length in the literature, whereas virtually no attention has been paid to either paired or joint multivariate non-nested tests. This paper examines how to use these tests empirically to compare alternative factor demand systems. The empirical application involves the classical Berndt-Khaled annual data set for the U.S. manufacturing sector over the period 1947-1971. A statistically adequate empirical specification is determined for each competing factor demand system, and the models are compared on the basis of multivariate paired and joint non-nested procedures. Finally, some practical suggestions are given for purposes of model selection and testing of systems of equations in applied research.

1. INTRODUCTION

Empirical factor demand analysis typically involves making a choice from among several competing non-nested functional forms. Each of the commonly used factor demand systems, such as Translog, Generalized Leontief, Quadratic and Symmetric Generalized McFadden, can provide a valid and useful empirical description of the underlying production structure of the multi-input neoclassical firm.

A common feature of flexible functional forms is that they are non-nested (or separate). Thus, given two or more systems of factor demands, it is not possible to obtain one system by imposing suitable parametric restrictions on the other(s). Moreover, as there is no a priori theory suggesting that the specification of one system should be preferred over another, it is necessary to choose from among the competing models using empirical considerations.

The important task of model determination can be accomplished using a formal non-nested testing procedure. Paired and joint univariate non-nested tests of a null model against both single and multiple alternatives have been discussed at length in the literature. However, virtually no attention has been paid to either paired or joint multivariate non-nested tests. This paper compares alternative factor demand systems using paired and joint multivariate non-nested tests.

As the outcome of a non-nested test is influenced by the type of misspecification affecting the competing models, it is essential to investigate the performance of each factor demand system against real data. The empirical application presented is very popular in the applied production literature, and contains annual data on aggregate output of U.S. manufacturing industries, and prices and quantities for a capital-labour-energy-materials (KLEM) technology over the period 1947-1971 (see Berndt and Khaled, 1979).

This paper reports empirical results from estimation and diagnostic testing of competing factor demand systems, namely Translog, Quadratic, Generalized Leontief and Symmetric Generalized McFadden (for further details, see Diewert and Wales, 1987). Multivariate paired and joint non-nested tests are applied to compare the alternative factor demand systems. The empirical results are interpreted for each system, and some guidelines are given for purposes of model selection and testing of systems of equations in applied research.

2. EMPIRICAL EVIDENCE ON FACTOR DEMAND SYSTEMS

Attention is focused on the four most widely used flexible functional forms in the context of cost function estimation: Translog, Quadratic, Symmetric Generalized McFadden and Generalized Leontief. As is customary in applied factor demand analysis, the cross-equation symmetry restrictions are maintained for each model. Several techniques are available for imposing the appropriate curvature conditions on the cost function (see Morey, 1986). In this paper, we have left unconstrained the matrix of second-order partial derivatives of the cost function with respect to factor prices in each model, and have checked ex post if the negative semi-definiteness of the Hessian of the cost function is satisfied over the sample period.

The systems of factor demands are estimated using Berndt and Khaled's (1979) classical annual data set for the U.S. manufacturing sector over the period 1947-1971. It is assumed that U.S. manufacturing can be described by a regular aggregate production function relating the flows of gross output Y to the services of four inputs, namely capital (K), labour (L), energy (E) and materials (M). Corresponding to such a production function, there exists a dual cost function summarizing all the characteristics of the representative firm's technology.

When output quantity and input prices are exogenous, the dual cost function can be written as:

$$C = C(Y, P_k, P_l, P_e, P_m, t) \qquad (2.1)$$

where $C(\cdot)$ represents total input costs, P_i, i=K,L,E,M, are the factor prices, and t is an index of technical progress.

For purposes of empirical implementation, the existence of random errors in the cost minimizing behaviour of the firm is such that each equation in each demand system has an additive disturbance term which reflects the firm's errors in deciding the optimal level of inputs. First-order serial correlation for each system is accommodated using a Cochrane-Orcutt transformation for each equation (see, e.g., Berndt, 1991, pp. 476-9). The estimated single-equation serial correlation coefficients have been used to estimate the system.

The Translog system (TLG) comprises the cost equation (2.1) and three share equations for labour, energy and materials, in addition to the linear homogeneity restrictions and symmetry conditions. It is well known that, for n factors, only n-1 share equations are estimated because the four cost shares sum to unity, so that the sum of the disturbances across the n=4 equations is zero for each observation. Consequently, the covariance matrix is singular and non-diagonal (Berndt and Wood, 1975, p. 261). The disturbance from one equation is omitted and the vector u comprising the disturbances of the remaining share equations and the cost function is specified as a multivariate normal distribution with $E(u)=0$ and $E(uu')=\Omega$, where Ω is constant over time (Diewert and Wales, 1987, p. 58).

The Quadratic demand system is given by the cost equation and the three demand equations for labour, energy and materials. The reason for excluding the capital equation is that, since linear homogeneity in prices cannot be imposed parametrically, a normalization with respect to an arbitrarily chosen factor price is required. QDR denotes the Quadratic demand system with linear homogeneity in prices imposed, NHQDR1 denotes the same system with non-homogeneity, that is, when linear homogeneity in prices is not imposed, and NHQDR denotes the Quadratic demand system formed from the four demand equations for capital, labour, energy and materials without imposing linear homogeneity in prices.

Finally, the Symmetric Generalized McFadden demand system (SGM) is formed from the four factor demands, together with appropriate identifying restrictions. The Generalized Leontief model (GLT) of factor demands is derived using Shephard's lemma. In both the SGM and GLT specifications, the dependent variables are input levels divided by output, as this makes the assumption of homoskedasticity of the disturbances more plausible. The cost function is not estimated since it does not contain any additional information.

The factor demand systems are estimated with the multivariate least squares routine Lsq implemented in Tsp 4.4 (for details, see Hall, Cummins and Schnake, 1997). Linear disembodied technical change is accommodated by the presence of linear and quadratic trends in the estimated equations. In particular, the results indicate the absence of both first-order serial correlation and heteroskedasticity, and the curvature properties of the firm's cost function are satisfied for each functional form.

Factor demand systems are typically used to calculate indicators which can be useful for describing the production structure of an economic sector. For each estimated model, the mean values of input demand elasticities with respect to input

prices and output over the period 1948-1971 are calculated. The magnitudes and signs of these elasticities depend crucially on the selected model. This is particularly true for the price elasticities of capital, which are roughly comparable for GLT, SGM, NHQDR and TLG, but appear quite different for NHQDR1 and QDR, that is, for the Quadratic functional form where the demand for capital is not directly estimated. In general, direct price elasticities are negative and output elasticities are positive, as suggested by theory, and the cross-price elasticities are all below one in absolute value.

From their signs, it is possible to obtain information about factor substitution and complementarity, which is also not independent of the functional form. For example, capital and energy are complements, according to GLT, NHQDR1, SGM and TLG, but substitutes on the basis of NHQDR and QDR. Capital and labour are substitutes for all models, except for NHQDR and NHQDR1. Labour and materials are complements according to only GLT, SGM and NHQDR1. Energy and materials are substitutes in all systems, with the exception of NHQDR1. Finally, materials and labour are complements for GLT, NHQDR1, SGM and TLG, whereas they are substitutes for NHQDR and QDR. In summary, all estimated systems seem to offer plausible interpretations of the production structure of the US manufacturing sector over the period 1948-1971, but the interpretations depend on the chosen specification. Systems QDR and NHQDR suggest the existence of substitution and complementarity relationships among the factors, which do not agree, in general, with the indications of the other models.

3. EMPIRICAL EVIDENCE ON MULTIVARIATE NON-NESTED TESTS

Alternative factor demand systems based on competing non-nested flexible functional forms were estimated in Section 2. Each model was statistically adequate and captured the relevant features of the data. In addition, the conclusions drawn in terms of price and output elasticities were not unique, and depended crucially on the chosen model. Thus, economic theory is of little assistance in discriminating among the competing models, and the empirical evidence suggests that the in-sample performance of each model is acceptable. Moreover, the choice of model has important implications for economic analysis. In this case, non-nested testing procedures can provide useful additional information. The factor demand systems of Section 2 are compared in this section on the basis of multivariate paired and joint non-nested tests.

The results of preliminary systems RESET tests for each competing model are calculated but not reported. These tests are calculated by adding the corresponding squared fitted values to each equation of the system and by testing their joint significance using a Wald statistic. Two versions of the systems RESET test are presented. The first test is calculated under the condition that the coefficients of the squared fitted values in each equation are different, leading to a Wald statistic with a $\chi^2(n)$ distribution, where n=4 is the number of equations in the system. The second version of the test is based on the condition that the coefficients of the squared fitted values in each equation are identical, and is a Wald test with a $\chi^2(1)$ distribution. Rejection of the null hypothesis of correct model specification by the $\chi^2(n)$-RESET test is interpreted as misspecification of at least one equation of the system, but not necessarily of the whole system, whereas rejection of the null hypothesis by the $\chi^2(1)$-RESET test indicates that the system itself is misspecified.

The results show that NHQDR and NHQDR1 are rejected at the 1% significance level by both the $\chi^2(4)$- and $\chi^2(1)$-RESET tests, and QDR is rejected by both versions of the test, but at different levels of significance. Specification TLG is rejected at the 1% significance level by the $\chi^2(4)$-RESET test, but is not rejected by the $\chi^2(1)$-RESET test. This is a contradiction, since the test indicates that functional form misspecification affects a sub-set of the system, but that the system itself does not suffer from misspecification. Finally, GLT and SGM are rejected only at the 5% significance level and only by the $\chi^2(4)$-RESET test. Thus, the systems RESET tests are incapable of determining a single model which performs best, although NHQDR and NHQDR1 appear to be more problematic than the others. This last evidence is in line with the empirical results of Section 2.

Table 1 shows the results obtained by comparing non-nested systems of equations with the same dependent variables, namely SGM and GLT on the one hand, and QDR and NHQDR1 on the other. The three different paired non-nested tests used in the empirical application are the J, P0 and P1 tests (see Davidson and MacKinnon, 1981). Each test is presented in two versions, with the $\chi^2(4)$-version being based on the condition that the coefficients are different across equations, whereas the $\chi^2(1)$-version imposes equality across the four equations. The interpretation of the non-nested tests is analogous to those of the systems RESET tests. Rejection of the null hypothesis of correct specification by the $\chi^2(4)$-test is interpreted as misspecification of at least one equation of the

system, but not necessarily of the whole system, against the chosen non-nested alternative. Conversely, a rejection of the null hypothesis by the $\chi^2(1)$-test indicates that the system itself is rejected against its non-nested counterpart.

The SGM system is rejected against GLT at the 1% significance level by the $\chi^2(4)$-version of each of the J, P0 and P1 tests. Each non-nested test suggests that there is a problem in at least one equation of the SGM system when compared with its GLT counterpart. The empirical evidence is mixed when the $\chi^2(1)$-version of the test is considered as, in this case, SGM is rejected at the 1% level only by the P0 test, whereas the J test rejects SGM at the 5% level, and the P1 test suggests non-rejection of SGM. In particular, the P1 test denotes an inconsistency as the $\chi^2(1)$-version implies that SGM as a whole is correctly specified as compared with its GLT counterpart. When the roles of the null and alternative are reversed, GLT is rejected at the 5% level by the $\chi^2(4)$-version of the J and P0 tests, whereas the P1 test rejects GLT at the 1% level. The behaviour of the J and P1 tests is contradictory, as the $\chi^2(1)$-version of these tests suggests that there are no problems with the specification of the GLT system as a whole, whereas the $\chi^2(1)$-version of the P0 test rejects the null at the 5% level. In summary, the multivariate paired non-nested tests suggest that both the SGM and GLT specifications suffer from problems of misspecification.

In Table 1, the second pair of competing models is given by QDR and NHQDR1. When QDR is the null hypothesis, it is strongly rejected by all three non-nested tests, under both the $\chi^2(4)$- and $\chi^2(1)$-versions. However, when NHQDR1 is the null, only the P1 test strongly rejects NHQDR1, regardless of which version of the test is used. The J test marginally rejects NHQDR1 with the $\chi^2(4)$-version, whereas it does not reject the null with the $\chi^2(1)$-version. The P0 test does not reject NHQDR1 at all. In this case, the results from these multivariate paired non-nested tests are interpreted as indirect evidence against the hypothesis of linear homogeneity in input prices.

Table 2 reports the results from the comparison of pairs of systems of equations with different dependent variables, where the competing pairs of systems are TLG and QDR, SGM and NHQDR, GLT and NHQDR, and TLG and NHQDR1. The multivariate non-nested tests used are the PE test (see MacKinnon et al., 1983) and the BEM test (see Bera and McAleer, 1989).

Both the $\chi^2(4)$- and $\chi^2(1)$-versions of the tests are presented. In general, all competing models are rejected by all tests, although there are a few interesting cases. When TLG and QDR are compared using the $\chi^2(1)$-version of the PE test, neither is rejected. Moreover, the $\chi^2(1)$-version of both the PE and BEM tests is unable to reject TLG against the alternative system NHQDR1. One possible interpretation is that the $\chi^2(1)$-version of the test is too restrictive to detect specification problems which are likely to affect only a sub-set of each system. A similar comment applies in testing TLG against NHQDR1 using the $\chi^2(1)$-version of both the PE and BEM tests.

Table 3 presents the empirical results obtained by using the multivariate joint BAM test (see Barten and McAleer, 1997). The three competing systems are NHQDR, SGM and GLT, and each null model is tested jointly against two alternatives. Four versions of the BAM test are available, according to the restrictions placed on the alternatives and on the equations. If the coefficients are unrestricted, this yields the $\chi^2(8)$-version of the BAM test, with four parameters across each of two alternative systems. The $\chi^2(4)$-version is obtained by using the same coefficient for the two non-nested alternatives across each of the four equations. The $\chi^2(2)$-version is obtained by using the same coefficient for each equation across each of two non-nested alternatives. Finally, the $\chi^2(1)$-version of the BAM test is obtained by using the same coefficient across four equations and two alternatives. Notice that the $\chi^2(8)$- and $\chi^2(2)$-versions do not impose restrictions across the alternative models, whereas the $\chi^2(4)$- and $\chi^2(1)$-versions do impose cross-alternative restrictions.

In general, all three models are strongly rejected, regardless of which version of the test is used. As before, there are a few cases worth highlighting. When the $\chi^2(2)$-version of the BAM test is used, TLG is not rejected against NHQDR1 and QDR jointly. Thus, the two alternative systems, each considered as a whole, do not add useful information to the TLG null model as a whole. When the $\chi^2(1)$-version of the test is used, neither QDR nor TLG is rejected against the other two models jointly. In this case, the added information given by a linear combination of both equations and alternatives is empirically irrelevant. Finally, if the $\chi^2(4)$-version of the test is considered, the TLG system is rejected only at the 5% significance level, whereas all other models are rejected at the 1% level. In summary, these results suggest that the information contained in a linear combination of the corresponding equations across the two alternatives is statistically important in rejecting each model against two non-nested alternatives jointly, at least at the 5% level.

4. CONCLUSION

The key points of this paper can be summarized as follows. Alternative factor demand systems have been presented using some of the most popular flexible functional forms, namely Translog, Quadratic, Generalized Leontief and Symmetric Generalized McFadden. Each system has been estimated using Berndt and Khaled's (1979) classical annual data set for the U.S. manufacturing sector over the period 1947-1971. The important task of model determination has been accomplished using a formal non-nested testing procedure. Multivariate extensions of some well-known and pedagogically appealing univariate paired non-nested tests (namely, the J, P0 and P1 tests of Davidson and MacKinnon, 1981; the PE statistic of MacKinnon, White and Davidson, 1983; and the Bera and McAleer (1989) test), and the multivariate joint non-nested test of Barten and McAleer (1997), have been applied to compare alternative factor demand systems. Preliminary systems RESET tests for each competing model were also discussed. Systems RESET tests and multivariate paired non-nested tests were each presented in two versions, namely $\chi^2(4)$ and $\chi^2(1)$. Four versions of the multivariate joint non-nested test were developed, namely $\chi^2(8)$, $\chi^2(4)$, $\chi^2(2)$ and $\chi^2(1)$.

The main results are as follows. Each model has been shown to be statistically adequate and to capture the relevant features of the data. In addition, the conclusions drawn in terms of price and output elasticities were not unique, and depended crucially on the chosen model. Systems RESET tests were incapable of determining a single model which performs best, although NHQDR and NHQDR1 appeared to be more problematic than the others. The multivariate paired extensions of the J, P0 and P1 tests suggested that both the SGM and GLT specifications suffered from problems of misspecification. When the same tests were used to compare QDR and NHQDR1, the results were interpreted as indirect evidence against the hypothesis of linear homogeneity in input prices. Systems involving different dependent variables were compared using the multivariate non-nested PE and BEM tests. In general, all competing models were rejected by all tests, although there were a few interesting cases. When TLG and QDR were compared using the $\chi^2(1)$-version of the PE test, neither was rejected. Moreover, the $\chi^2(1)$-version of both the PE and BEM tests was unable to reject TLG against the alternative system NHQDR1. Finally, the multivariate joint BAM test was used to compare systems NHQDR, SGM and GLT. In general, the results suggested that the information contained in a linear combination of the corresponding equations across the two alternatives was statistically important in rejecting each model against two non-nested alternatives jointly, at least at the 5% level.

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Table 1: Multivariate *paired* non-nested tests: J, P0 and P1

H_0	H_1	$\chi^2(4)$	$\chi^2(1)$
J Test			
SGM	GLT	16.103**	6.157*
		(0.003)	(0.013)
GLT	SGM	12.818*	3.0005
		(0.012)	(0.083)
QDR	NHQDR1	254.161**	250.495**
		(0.000)	(0.000)
NHQDR1	QDR	9.802*	1.452
		(0.044)	(0.228)
P0 Test			
SGM	GLT	25.140**	7.089**
		(0.000)	(0.008)
GLT	SGM	11.843*	3.904*
		(0.019)	(0.048)
QDR	NHQDR1	201.006**	196.511**
		(0.000)	(0.000)
NHQDR1	QDR	8.223	1.513
		(0.084)	(0.219)
P1 Test			
SGM	GLT	34.884**	3.167
		(0.000)	(0.075)
GLT	SGM	23.564**	1.457
		(0.000)	(0.227)
QDR	NHQDR1	2191.842**	2172.844**
		(0.000)	(0.000)
NHQDR1	QDR	207.978**	203.181**
		(0.000)	(0.000)

Notes: * denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 1% significance level; p-values are given in parentheses.

Table 2: Multivariate *paired* non-nested tests: PE and BEM

H_0	H_1	$\chi^2(4)$	$\chi^2(1)$
PE Test			
TLG	QDR	43.533**	0.134
		(0.000)	(0.714)
QDR	TLG	22.394**	1.486
		(0.000)	(0.223)
SGM	NHQDR	114.973**	103.424**
		(0.000)	(0.000)
NHQDR	SGM	21.583**	9.397*
		(0.000)	(0.002)
NHQDR	GLT	40.532**	13.489**
		(0.000)	(0.000)
GLT	NHQDR	130.980**	112.255**
		(0.000)	(0.000)
TLG	NHQDR1	23.299**	0.488
		(0.000)	(0.485)
NHQDR1	TLG	34.810**	8.908**
		(0.000)	(0.003)
BEM Test			
TLG	QDR	10.419*	2.880
		(0.034)	(0.090)
QDR	TLG	44.825**	43.371**
		(0.000)	(0.000)
SGM	NHQDR	74.299**	71.625**
		(0.000)	(0.000)
NHQDR	SGM	30.161**	14.297**
		(0.000)	(0.000)
NHQDR	GLT	45.783**	17.621**
		(0.000)	(0.000)
GLT	NHQDR	61.417**	52.231**
		(0.000)	(0.000)
TLG	NHQDR1	51.168**	0.144
		(0.000)	(0.705)
NHQDR1	TLG	307.541**	50.291**
		(0.000)	(0.000)

Notes: * denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 1% significance level; p-values are given in parentheses.

Table 3: Multivariate *joint* non-nested test: BAM

H_0	H_1	H_2	$\chi^2(8)$	$\chi^2(2)$	$\chi^2(4)$	$\chi^2(1)$
NHQDR	SGM	GLT	136.063**	22.387**	37.245**	15.928**
			(0.000)	(0.000)	(0.000)	(0.000)
SGM	NHQDR	GLT	102.234**	71.896**	74.236**	71.601**
			(0.000)	(0.000)	(0.000)	(0.000)
GLT	NHQDR	SGM	74.306**	52.565**	61.389**	52.239**
			(0.000)	(0.000)	(0.000)	(0.000)
QDR	NHQDR1	TLG	52.809**	42.687**	26.982**	0.365
			(0.000)	(0.000)	(0.000)	(0.546)
TLG	NHQDR1	QDR	72.120**	3.712	11.393*	1.121
			(0.000)	(0.156)	(0.022)	(0.290)
NHQDR1	TLG	QDR	377.083**	51.890**	23.186**	9.582**
			(0.000)	(0.000)	(0.000)	(0.002)

Notes: * denotes rejection of the null hypothesis H_0 against the multiple alternatives H_1 and H_2 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the multiple alternatives H_1 and H_2 at the 1% significance level; p-values are given in parentheses.